Comprehensive Derivation of a Density Limit of the Evolution of Space

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Abstract

No density can be larger than the Planck density. The time evolution of the actual light horizon should be traced back until the Planck length is reached. However there arises a problem, as in the framework of general relativity theory, GRT, that length is only reached at the density much larger than the Planck density. We investigate the Planck scale, the evolution of space according to the Friedmann Lemaitre equation and the resulting density limit by using EXCEL in a graphic manner. So, we achieve a comprehensive understanding based on our own activity. Additionally, we outline a possible solution of that problem.

1. Introduction

Since the Big Bang the universe expands (Einstein 1917, Wirtz 1922, Hubble 1929, Friedmann 1922, Lemaître 1927, Planck 2018). Usually that expansion is modeled in the framework of general relativity theory, GRT (Einstein 1915). In the early universe, the density was very high. So quantum physics is essential. In particular, there is an upper limit of the density, the Planck density $\rho_P = 5{,}155 \cdot 10^{96}$ kg/m^3 and the corresponding length scale of the Planck length $L_P = 1{,}616 \cdot 10^{-35}$ m.

However, in the model in the framework of the GRT, the density exceeds the Planck density at a relatively large length scale of 0.04 mm, and so it doesn´t achieve the Planck length.

1.1. Students

We elaborate the occurrence of this problem in more detail. This problem is solved in a parallel reports by Schöneberg and Carmesin as well as by Carmesin. Moreover, the solution has been elaborated with other scientific tools in (Carmesin 2017, Carmesin 2018a-d, Carmesin 2019a-b, Carmesin 2020a-b).

The present project has been worked out in a research club with students ranging from classes 9 to In the project, the students develop many process related competences such as modeling, epistemology, computer experiments, mathematics, numerical computations and communication (Niedersächsisches Kultusministerium 2017). Additionally, the students use their present competences in order to derive additional insights and competences. This provides a high efficiency of learning (Hattie 2009). Furthermore, the students develop insights in an elementary manner on their own. Such learning is efficient in science education (Kircher 2001). Moreover the students presented their results at a public astronomy evening in the Aula of their school, full of interested visitors. Thereby the trained their communication skills in a very challenging manner. In summary, the students achieve a comprehensive understanding based on their own activity.

2. Derivation of the dynamics based on the GRT

2.1 Used variables

We use various variables in the derivation of a suitable formula:

x = describes the light horizon we want to calculate and study more closely.
t = means the time.
1 = is defined as the largest possible value of x here and describes the present time. The smaller t becomes, the further back in time it is.
For the space expansion we use the variable a. Interesting is the relation between space expansion a and time t.
The last variable is ρ. We also look at this in relation to time to determine by what density the space has changed over time.

2.2 Used physical quantities

The density of the universe is composed of the density of dark energy, the density of matter and the density of radiation. The density parameter Ω_Λ describes the proportion of dark energy in the universe which is 68.47% of the total energy density in the universe. Ω_m describes the proportion of energy density in matter, which amounts to 31.53% in our universe, and Ω_r represents the portion of radiation, which amounts to 0.009265% (Planck 2018, Carmesin 2019a). $\rho_{cr,t0}$ means the critical density at time 0, today. The constant G describes the gravitational constant, which is $6.67384 \cdot 10^{-11}$ m^3/kg·s^2. It describes the strength of the gravitation. In the second part of the calculation, we also include the scaling radius a, which is $4.15 \cdot 10^{26}$ m, it describes today's visible range including the mass $2.593 \cdot 10^{54}$ kg.

2.3 Integration of the differential equation, DEQ

We want to calculate the dynamics of the light horizon. For this we need the following basic formulas:

The density consists of three components:

$$\rho_r + \rho_m + \rho_v = \rho_{cr,t_0} \cdot x^{-4} \cdot (\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r) \qquad \{1\}$$

The critical density is:

$$\rho_{cr,t_0} = \frac{3 \cdot H_0^2}{8\pi \cdot G} \qquad \{2\}$$

The DEQ is the Friedmann Lemaître equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi \cdot G}{3} \cdot (\rho_r + \rho_m + \rho_v) \qquad \{3\}$$

Using the Leibniz calculus,

$$\dot{a} = \frac{da}{dt}$$

we get:

$$\frac{\left(\frac{da}{dt}\right)^2}{a^2} = \frac{8\pi \cdot G}{3} \cdot (\rho_r + \rho_m + \rho_v) \qquad \{4\}$$

Extension of the fracture with $1/a_0$:

$$\frac{\left(\frac{da/a_0}{dt}\right)^2}{a^2/a_0} = \frac{8\pi \cdot G}{3} \cdot (\rho_r + \rho_m + \rho_v) \qquad \{5\}$$

Application of the definition $x = a/a_0$:

$$\frac{\left(\frac{dx}{dt}\right)^2}{x^2} = \frac{8\pi \cdot G}{3} \cdot (\rho_r + \rho_m + \rho_v) \qquad \{6\}$$

Application of the equation {2}:

$$\frac{\left(\frac{dx}{dt}\right)^2}{x^2} = \frac{8\pi \cdot G}{3} \cdot \rho_{cr,t_0} \cdot x^{-4} \cdot (\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r) \qquad \{7\}$$

Application of the equation {3}:

$$\frac{\left(\frac{dx}{dt}\right)^2}{x^2} = H_0^2 \cdot x^{-4} \cdot (\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r) \qquad \{8\}$$

Multiplying on both sides of the equation by $x^2 \cdot dt^2$:

$$dx^2 = dt^2 \cdot H_0^2 \cdot x^{-2} \cdot (\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r) \qquad \{9\}$$

taking the root:

$$dx = dt \cdot H_0 \cdot x^{-1} \cdot \sqrt{(\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r)} \qquad \{10\}$$

Application of : $dt \cdot H_0 = d\tau$:

$$dx = d\tau \cdot x^{-1} \cdot \sqrt{(\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r)} \qquad \{11\}$$

Now we bring all terms with x on one side, in order to separate the variables and then form an integral $\int$ dx.

$$\frac{x}{\sqrt{(\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r)}} dx = d\tau \qquad \{12\}$$

We integrate:

$$\int_0^x \frac{x}{\sqrt{(\Omega_\Lambda \cdot x^4 + \Omega_m \cdot x + \Omega_r)}} dx = \int_0^\tau d\tau \qquad \{13\}$$

2.4 Spreadsheet

The equation {13} can now be used to create a table steering calculation. For Ω_Λ we use 0.6847, for Ω_m 0.3153 and for Ω_r 0.00009265.

With the help of Wolfram Alpha, we now calculate the individual integrals. Fig. 1 shows the procedure using three different integration limits.

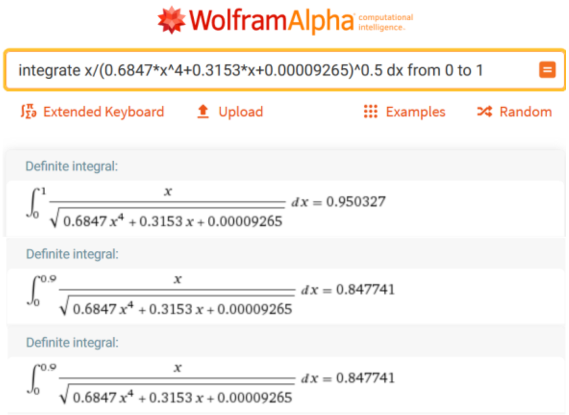

Fig.1: Solving the integral using Wolfram Alpha.

We record all the results in a table, which we expand to include the space expansion a and density ρ later. To be able to calculate a and ρ now, we use the following formulas, see above:

$$a = x \cdot 4{,}15 \cdot 10^{26} m$$

$$\rho = \frac{2.593 \cdot 10^{54}}{4 \cdot \frac{\pi}{3} \cdot a^3} \cdot \left(0.6847 + \frac{0.3153}{x^3} + \frac{0.00009265}{x^4}\right)$$

So, for the calculation of the density, we use the different density fractions in the universe again, as this makes the calculations more precise.

2		a	= 4.15E+26	
3		M	2.593E+54	
4		rho	=M/(4*PI()/3*C10^3)	
5		ΩΛ	0.6847	
6		ΩM	0.3135	
7		ΩR	0.00009265	
8				
9	t	x	a	rho
10	0.950327	1	=B10*a	=M/(4*PI()/3*C10^3)
11	0.899675	0.95	=B11*a	=rho*(ΩΛ+ΩM/B11^3+ΩR/B11^4)
12	0.847741	0.9	=B12*a	=rho*(ΩΛ+ΩM/B12^3+ΩR/B12^4)
13	0.794586	0.85	=B13*a	=rho*(ΩΛ+ΩM/B13^3+ΩR/B13^4)
14	0.740307	0.8	=B14*a	=rho*(ΩΛ+ΩM/B14^3+ΩR/B14^4)
15	0.685039	0.75	=B15*a	=rho*(ΩΛ+ΩM/B15^3+ΩR/B15^4)
16	0.628971	0.7	=B16*a	=rho*(ΩΛ+ΩM/B16^3+ΩR/B16^4)
17	0.572343	0.65	=B17*a	=rho*(ΩΛ+ΩM/B17^3+ΩR/B17^4)
18	0.515459	0.6	=B18*a	=rho*(ΩΛ+ΩM/B18^3+ΩR/B18^4)
19	0.458681	0.55	=B19*a	=rho*(ΩΛ+ΩM/B19^3+ΩR/B19^4)
20	0.402433	0.5	=B20*a	=rho*(ΩΛ+ΩM/B20^3+ΩR/B20^4)
21	0.347195	0.45	=B21*a	=rho*(ΩΛ+ΩM/B21^3+ΩR/B21^4)
22	0.293491	0.4	=B22*a	=rho*(ΩΛ+ΩM/B22^3+ΩR/B22^4)
23	0.192958	0.3	=B23*a	=rho*(ΩΛ+ΩM/B23^3+ΩR/B23^4)
24	0.105665	0.2	=B24*a	=rho*(ΩΛ+ΩM/B24^3+ΩR/B24^4)
25	0.0373771	0.1	=B25*a	=rho*(ΩΛ+ΩM/B25^3+ΩR/B25^4)
26	0.0131678	0.05	=B26*a	=rho*(ΩΛ+ΩM/B26^3+ΩR/B26^4)
27	0.00114575	0.01	=B27*a	=rho*(ΩΛ+ΩM/B27^3+ΩR/B27^4)

Fig.2: Formulas for calculating a and ρ.

In Fig. 2, the formulae for calculating the light horizon a and the density ρ are shown in the table. With the help of these, we completed the table and finally received a finished spreadsheet (Fig. 3)

3. Time evolution of radius and density

From the x and t values, we make two diagrams in order to better illustrate the radius per time. We recognize these two x(t) diagrams in Figs. 4 and 5, Fig. 4 being linear and Fig. 5 logarithmic.

2		a	4.15E+26	
3		M	2.593E+54	
4		rho	8.661E-27	
5				
6	t	x	a	rho
7	0.95033	1	4.15E+26	8.66103E-27
8	0.89968	0.95	3.9425E+26	9.11629E-27
9	0.84774	0.9	3.735E+26	9.67742E-27
10	0.79459	0.85	3.5275E+26	1.03784E-26
11	0.74031	0.8	3.32E+26	1.12658E-26
12	0.68504	0.75	3.1125E+26	1.24058E-26
13	0.62897	0.7	2.905E+26	1.38951E-26
14	0.57234	0.65	2.6975E+26	1.58785E-26
15	0.51546	0.6	2.49E+26	1.85791E-26
16	0.45868	0.55	2.2825E+26	2.23526E-26
17	0.40243	0.5	2.075E+26	2.77896E-26
18	0.3472	0.45	1.8675E+26	3.59177E-26
19	0.29349	0.4	1.66E+26	4.86307E-26
20	0.19296	0.3	1.245E+26	1.07171E-25
21	0.10567	0.2	8.3E+25	3.47785E-25
22	0.03738	0.1	4.15E+25	2.74478E-24
23	0.01317	0.05	2.075E+25	2.19809E-23
24	0.00115	0.01	4.15E+24	2.81107E-21
25	4.70E-07	0.0001	4.15E+22	1.07553E-14
26	5.19E-11	1.00E-06	4.15E+20	8.05175E-07

Fig.3: Results in the spreadsheet.

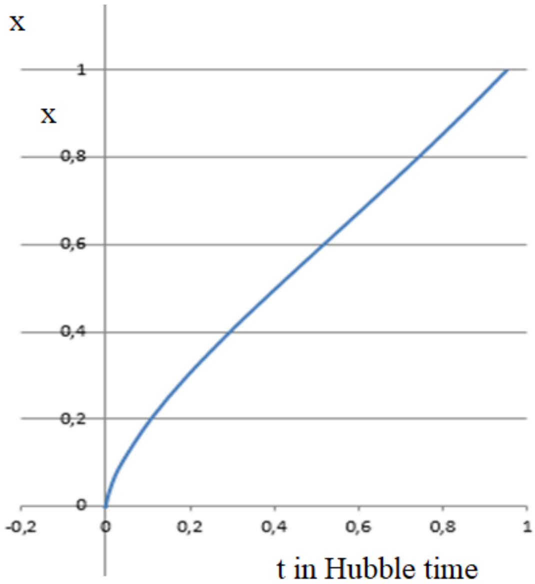

Fig.4: Time evolution of the light horizon.

The densities as a function of the time and the radius are investigated with a spreadsheet (Fig. 6), thereby the table in Fig. 3 is continued. We see (Fig. 6) that the Planck density is reached at a size of 0.000014524 m of the light horizon (green). At smaller values of the light horizon, the density would exceed the Planck density (red). However, this is not possible (see for instance Carmesin 2019a). So we conclude that the dynamics of the GRT is not complete, as the density would increase

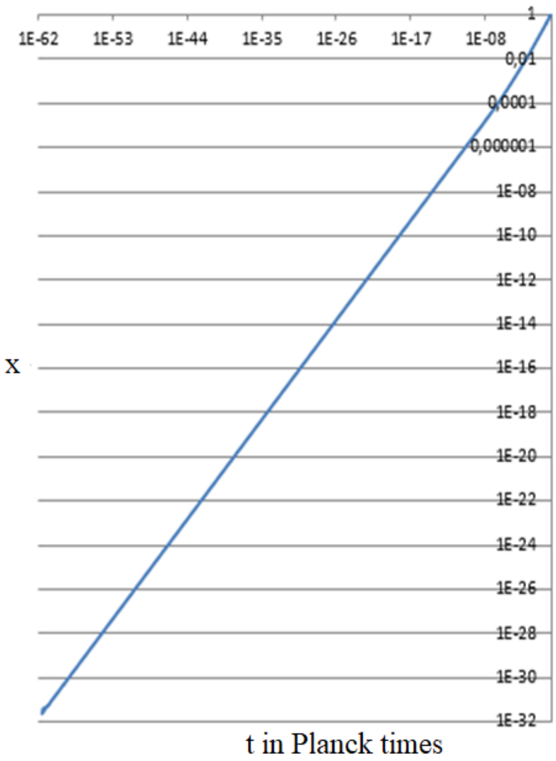

Fig.5: Time evolution of light horizon: Logarithmic scale

the Planck density in the early universe. So there is a density limit, at which the dynamics of the GRT is not applicable.

5,19E-43	1,00E-22	41500	8,02445E+57
5,19E-47	1,00E-24	415	8,02445E+65
5,19E-51	1,00E-26	4,15	8,02445E+73
5,19E-55	1,00E-28	0,0415	8,02445E+81
5,19E-59	1,00E-30	0,000415	8,02445E+89
1,29E-59	5,00E-31	0,0002075	1,28391E+91
2,07E-60	2,00E-31	0,000083	5,01528E+92
5,19E-61	1,00E-31	0,0000415	8,02445E+93
2,54E-61	7,00E-32	0,00002905	3,34213E+94
1,29E-61	5,00E-32	0,00002075	1,28391E+95
4,31E-62	4,00E-32	0,0000166	3,13455E+95
6,36E-62	3,50E-32	0,000014525	5,3474E+95
4,67E-62	3,00E-32	0,00001245	9,90672E+95
4,37E-62	2,90E-32	0,000012035	1,13455E+96
3,79E-62	2,70E-32	0,000011205	1,50994E+96
3,25E-62	2,50E-32	0,000010375	2,05426E+96
2,07E-62	2,00E-32	0,0000083	5,01528E+96
5,19E-63	1,00E-32	0,00000415	8,02445E+97
5,19E-67	1,00E-34	4,15E-08	8,0244E+105
5,19E-71	1,00E-36	4,15E-10	8,0244E+113
5,19E-75	1,00E-38	4,15E-12	8,0244E+121
5,19E-79	1,00E-40	4,15E-14	8,0244E+129
5,19E-83	1,00E-42	4,15E-16	8,0244E+137
5,19E-87	1,00E-44	4,15E-18	8,0244E+145
5,19E-91	1,00E-46	4,15E-20	8,0244E+153
5,19E-95	1,00E-48	4,15E-22	8,0244E+161
5,19E-99	1,00E-50	4,15E-24	8,0244E+169
5,19E-103	1,00E-52	4,15E-26	8,0244E+177
5,19E-107	1,00E-54	4,15E-28	8,0244E+185
5,19E-111	1,00E-56	4,15E-30	8,0244E+193
5,19E-115	1,00E-58	4,15E-32	8,0244E+201
5,19E-119	1,00E-60	4,15E-34	8,0244E+209
5,19E-121	1,00E-61	4,15E-35	8,0244E+213

Fig.6: Density (right column) as a function of the time in Hubble times (first column) and of the light horizon in m (third column).

4. Discussion

We investigated then time evolution of the light horizon and of the density of the universe in the framework of the GRT and with the corresponding DEQ. As a result, we see that the DEQ describes the dynamics in the range from the actual density to the Planck density. Thereby the lengths are calculated in the range from the actual light horizon $4.15 \cdot 10^{26}$ m to 0.000014 m. The DEQ fails to model the lengths in the range from 0.000014 m to the Planck length $1.616 \cdot 10^{-35}$ m. So the following factor is not explained by the GRT: $q = 0.000014/1.616 \cdot 10^{-35}$. This factor amounts to $q = 8.71.616 \cdot 10^{29}$. This missing dynamical factor has been estimated by Guth (1981). This factor can be explained by the folding of the space to higher dimensions (see Schöneberg and Carmesin in a parallel report and Carmesin 2017, Carmesin 2018a-d, Carmesin 2019a-b, Carmesin 2020a-b). Note that higher dimensions have already observed experimentally (Lohse 2018, Zilberberg 2018).

The project shows how students in a research club in classes ranging from 9 to 12 can derive significant results by themselves. Thereby they use and improve their competences efficiently (Hattie 2009) in a constructive manner (Kircher 2001). So such projects are very useful in science education.

5. Literature

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